4.6 VARIATION OF PARAMETERS

IF y_1, y_2 ARE L.I. SOLUTIONS OF $y'' + py' + qy = 0$

THEN $y_p = -y_1 \int \frac{g y_2}{W[y_1, y_2]}\,dx + y_2 \int \frac{g y_1}{W[y_1, y_2]}\,dx$

IS A SOL'N OF $y'' + py' + qy = g$

↳ GENERAL SOL'N

$$y = y_p + c_1 y_1 + c_2 y_2$$

GIVEN $y = \underbrace{\frac{\sin x}{\sqrt{x}}}_{y_1}$ IS A SOL'N OF $x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0$

FIND THE GENERAL SOL'N OF $x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 3x^{\frac{3}{2}} \sin x$

NEED TO FIND $y_2 \longrightarrow y_2 = v y_1$ REDUCTION OF ORDER 4.2

NEED TO FIND y_p

IN FACT, $y_2 = \frac{\cos x}{\sqrt{x}}$

$$g = \frac{3x^{\frac{3}{2}} \sin x}{x^2}$$

$$g = 3x^{-\frac{1}{2}} \sin x$$

CONT'D

$$y_p = -y_1 \int \frac{g y_2}{W}\,dx + y_2 \int \frac{g y_1}{W}\,dx \qquad \begin{aligned} y_1 &= x^{-\frac{1}{2}}\cos x \\ y_2 &= x^{-\frac{1}{2}}\sin x \end{aligned}$$

$$W = \begin{vmatrix} x^{-\frac{1}{2}}\cos x & x^{-\frac{1}{2}}\sin x \\ -\frac{1}{2}x^{-\frac{3}{2}}\cos x - x^{-\frac{1}{2}}\sin x & -\frac{1}{2}x^{-\frac{3}{2}}\sin x + x^{-\frac{1}{2}}\cos x \end{vmatrix}$$

$$= x^{-1}\cos^2 x - - x^{-1}\sin^2 x$$

$$= x^{-1}(\cos^2 x + \sin^2 x) = x^{-1}$$

$$1 - 2\sin^2 x = \cos 2x$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\int \sin^2 x\,dx = \tfrac{1}{2}\int (1 - \cos 2x)\,dx$$

$$= \tfrac{1}{2}\left(x - \tfrac{1}{2}\sin 2x\right)$$

$$= \tfrac{1}{2}x - \tfrac{1}{2}\sin x \cos x$$

$$y_p = -x^{-\frac{1}{2}}\cos x \int \frac{3\cancel{x^{-\frac{1}{2}}}\sin x \cdot \cancel{x^{-\frac{1}{2}}}\sin x}{\cancel{x^{-1}}}\,dx$$

$$+ x^{-\frac{1}{2}}\sin x \int \frac{3\cancel{x^{-\frac{1}{2}}}\sin x \cdot \cancel{x^{-\frac{1}{2}}}\cos x}{\cancel{x^{-1}}}\,dx$$

$$= -3x^{-\frac{1}{2}}\cos x \int \sin^2 x\,dx + 3x^{-\frac{1}{2}}\sin x \int \sin x \cos x\,dx$$

$$u = \sin x$$
$$du = \cos x\,dx \rightarrow \int u\,du$$

$$= -3x^{-\frac{1}{2}}\cos x\left(\tfrac{1}{2}x - \tfrac{1}{2}\sin x \cos x\right) + 3x^{-\frac{1}{2}}\sin x \cdot \tfrac{1}{2}\sin^2 x$$

$$= -\tfrac{3}{2}x^{\frac{1}{2}}\cos x + \underline{\tfrac{3}{2}x^{-\frac{1}{2}}\sin x}\cos^2 x + \underline{\tfrac{3}{2}x^{-\frac{1}{2}}\sin}^3 x$$

$$y_p = -\frac{3}{2}x^{\frac{1}{2}}\cos x + \cancel{\underbrace{\frac{3}{2}x^{-\frac{1}{2}}\sin x}_{\frac{3}{2}y_2}\underbrace{\left(\cos^2 x + \sin^2 x\right)}_{1}}$$

$$y_p = -\frac{3}{2}x^{\frac{1}{2}}\cos x$$

$$y = -\frac{3}{2}x^{\frac{1}{2}}\cos x + C_1 x^{-\frac{1}{2}}\cos x + C_2 x^{-\frac{1}{2}}\sin x$$

4.9 SYSTEMS OF LDE WITH CONSTANT COEF'S

eg. $\frac{dx}{dt} = x - y$

$\frac{dy}{dt} = y - 4x$

TANKS A, B ARE INTERCONNECTED

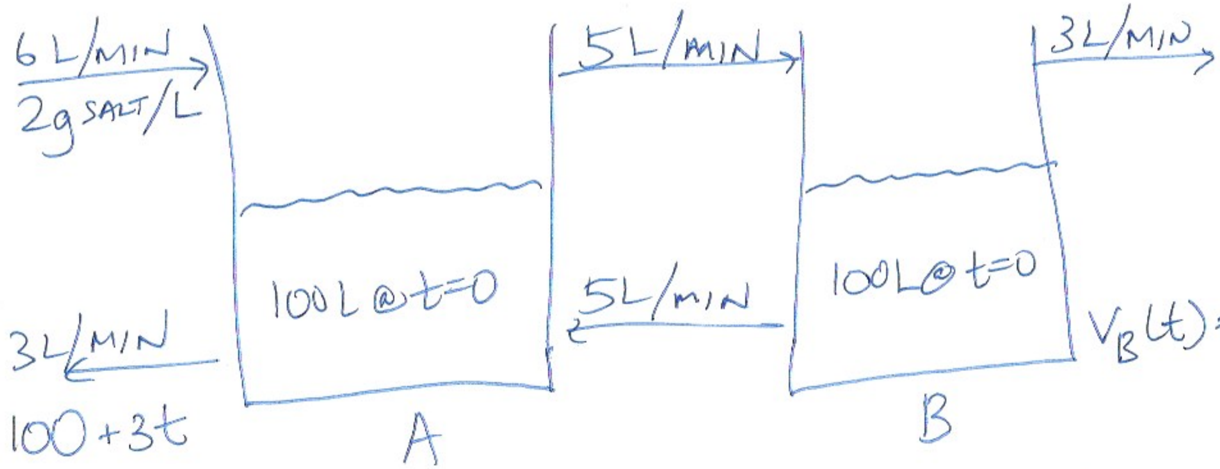
AT TIME $t=0$ (MINUTES), TANK A CONTAINS 100 L OF WATER

B 100 L OF WATER WITH 200 g OF SALT DISSOLVED

BRINE WITH A CONCENTRATION OF 2 g SALT/L STARTS FLOWING INTO TANK A AT 6 L/MINUTE

THE WELL MIXED SOLUTIONS IN TANKS A, B FLOW INTO EACH OTHER AT 5 L/MINUTE

IN ADDITION, SOLUTION IN TANK A + B. LEAVES THE SYSTEM AT 3 L/min



$V_A(t) = 100 + 3t$

$V_B(t) = 100 - 3t$

$A(t)$ = AMOUNT OF SALT IN TANK A AT TIME t

$B(t)$ = " " " B "

$$\frac{dA}{dt} = \text{RATE SALT INTO } A - \text{RATE SALT OUT OF } A$$

$$= \underbrace{6\,\frac{\cancel{L}}{\text{min}} \cdot \frac{2g}{\cancel{L}}}_{\text{IN FROM OUTSIDE SYSTEM}} + \frac{5L}{\text{min}} \cdot \frac{B(t)}{100-3t} - \left(\frac{(3+5)L}{\text{min}} \cdot \frac{A(t)}{100+3t} \right)$$

$$\frac{dA}{dt} = 12 + \frac{5B}{100-3t} - \frac{8A}{100+3t}$$

YOU FIGURE OUT THIS DE

$$\frac{dB}{dt} =$$